Mechanical Behavior of Closed Cell Plastic Foams Used as Cushioning Materials

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Synopsis

Two methods for the prediction of force–deformation curves of closed cell plastic foams at any strain rate from a limited number of experiments are described. These methods are based on the "modified Boltzman integral model" and on the "reference model." Both methods use constitutive equations and experimentally determined parameters. The modified Botlzman integral model uses data obtained in a limited number of stress–relaxation experiments while the reference model uses a very limited number of stress–relaxation and one force–deformation curve data. Both models predict well the force–deformation curves, the reference model providing somewhat better predictions.

INTRODUCTION

Plastic foams assume a growing range of engineering applications and are extensively used also as cushioning materials for the protection of fragile products during handling and transportation. By damping out vibration amplifications and mitigating shocks, the packaged items reach their final destination with much less damage.

In previous publications[1,2] an attempt was made to predict cushioning curves of plastic foams (to be used for cushion design) from their compressive behavior at relatively slow rates. The discrepancy between the calculated and experimentally determined curves has suggested that compressive properties obtained at high rates of deformation together with constitutive equations should enable more accurate predictions of the dynamic behavior of the plastic foams.

Ramon et al.[3] showed recently how phenomenological viscoelastic theories developed for bulk polymers could be adapted for the prediction of the mechanical properties and behavior of open cell plastic foams. It is the purpose of the present publication to demonstrate how these methods could also be applied to predict the mechanical properties of closed cell plastic foams.

THEORETICAL ANALYSIS

Rusch[4] described the compressive stress–strain curves of plastic foams by the following constitutive equation:

$$\sigma = E_f \cdot \epsilon \cdot Y(\epsilon) \tag{1}$$

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where σ is the compressive stress, E_f the apparent modulus of elasticity of the foam, ϵ the strain, and $Y(\epsilon)$ a strain function. Thus, Rusch expressed the compressive stress as a product of a linear elastic response ($E_f \cdot \epsilon$) and a nonlinear strain function $Y(\epsilon)$ that reflects the collapse of the foam cells during compression.

In the case of open cell foams, most of the air is forced out during compression of the foam. In closed cell foams, however, the air is unable to escape and is therefore compressed together with the foam matrix. The contribution of the entrapped air in closed cell foams to the compressive stress was approximated by Rusch[5] by adding an additional term in eq. (1) in the following form:

$$\sigma = E_f \cdot \epsilon \cdot Y(\epsilon) + P_a V_a \cdot \epsilon/(1 - V_p - \epsilon) \tag{2}$$

where P_a is the atmospheric pressure, V_a the volume fraction of closed cells, and V_p the volume fraction of the polymer in the foam. The last term in eq. (2) becomes negligible in the following cases: (a) a very stiff matrix; (b) large fraction of open cells ($0 < V_a \ll 1$); (c) very brittle matrix, resulting in easy fracture of cells and escape of gas.

Tsai[6] expressed the compressive force corresponding to the rupture of cells by the following equation:

$$F = m(d_f/d_p)^2 \cdot \dot{\epsilon}^n A_f/9 \tag{3}$$

where m is defined as the consistency of the foam, d_f and d_p as the foam and polymer density, respectively, $\dot{\epsilon}$ is the strain rate, n the power law index, and A_f the surface area of the foam.

It was shown[3,7,8] that the stress–strain relation at any rate of deformation can be calculated from data generated in stress–relaxation experiments using mathematical models that take into account the nonlinear viscoelastic behavior of plastic foams at large deformations. Hong et al.[9] showed that in stress–relaxation experiments time and strain effects could be separated. They[9] have also outlined the cases and criteria when this factorization is possible and have justified and stated that unlike in stress–relaxation, in constant compressive or tensile experiments this factorization is not necessarily applicable.

In many cases, the relaxation modulus E_r of elastomers and their foams can be described as a linear function of time on a logarithmic scale,[8–10] and parallel lines are obtained at different strains when plotting E as a function of time according to the following power law equation:

$$E_r(t) = Kt^{-n} \tag{4}$$

Factorization of time and strain have also been successfully applied to stress–relaxation data of polymeric foams used as cushioning materials.[3] The purpose in the latter case was to enable the prediction of stress–strain curves at different rates from a limited number of stress–relaxation experiments.

As was pointed out earlier, the stress–relaxation curves of most solid polymers and open cell foams, expressed as modulus vs. time, are generally parallel when generated at different strains. This is, however, not necessarily the case for foams with a large fraction of closed cells as was observed for a polyvinylacrylonitrile foam.[7] In the latter case the stress–relaxation curve may also be influenced by the compression and diffusion of gases into and through the cell walls in addition to its dependence on the stress–relaxation behavior of the matrix. Thus, the slope of the straight lines were found to depend also on the initial strain from which relaxation began. Nagy et al.[10] and Meinecke and Clark[7] described this relationship by the following equation:

$$n(\epsilon) = a + b\epsilon \tag{5}$$

where a and b are constants.

There is no obvious reason, however, why the slope should be linearly dependent on the strain and more complex expressions could also be obtained. Moreover, the parameter n may also be a function of the strain rate and not of the strain only. In such a case the relaxation modulus can be described by the following equation:

$$E_r(t, \epsilon, \dot{\epsilon}) = K_r t^{-n(\epsilon, \dot{\epsilon})} f(\epsilon) \tag{6}$$

Thus the stress–relaxation expression is more complex and the prediction of the load–deformation curves is more complicated. In constant deformation rate experiments it can be assumed that the strain ϵ is a linear function of time, namely,

$$\epsilon = \dot{\epsilon} t \tag{7}$$

For the analysis of the results we have assumed that when the effect of strain rate on the parameter n in eq. (6) is taken into account, the following modified Boltzman integral:

$$\sigma_r(\epsilon, t) = K_r \dot{\epsilon} f(\epsilon) \int_0^t (t - \theta)^{-n(\dot{\epsilon}\theta, \dot{\epsilon})} \, d\theta \tag{8}$$

could be used to predict the stress–strain curves of closed cell foams from relaxation experiments even if the lines of log E_r vs. log t, at different strains, are not parallel.

EXPERIMENTAL

Materials

A crosslinked, closed cell polyethylene (PE) foam and an expanded polystyrene (PS) foam were studied in the present work. Some of the properties of these foams are summarized in Table I. Cylindrical samples of a large enough diameter (to prevent collumnar buckling according to Kerstner[11]) were used. The minimum sample area A_{min} is given by the following equation:

$$A_{min} = (1.33 L_0)^2 \tag{9}$$

TABLE I
Characteristic Parameters and Properties of the Investigated Foams

Foam	Foam thickness (cm)	Density (g/cc)	Average cell size (mm)	Modulus of elasticity[a] (kg/cm^2)
PS	5	0.01, 0.02, 0.03	0.08	23.0
PE	3	0.041	0.3–0.7	4.97

[a]At a strain rate of $0.0033\ s^{-1}$.

where L_0 is the sample thickness. The measured Poisson ratios of the studied foams were very small and therefore stresses and strains were defined as "engineering" (based on the initial cross-sectional area, normally 100 cm^2).

Methods

Stress–relaxation and stress–strain curves, in compression, were obtained with a J. J. Lloyds, Model 5002, universal testing machine. In the stress–relaxation experiments the foam was compressed at rates in the range of $0.0033-1\ s^{-1}$ to the preset strain. The crosshead was then maintained at that strain and the force decay was monitored as a function of time. In the force–deformation experiments the samples were compressed at crosshead speeds of 1–45 cm/min in the J. J. Lloyds instrument and at speeds of 40–300 cm/min in a MTS 810 tester. All data acquisition and storage were performed with the aid of a Digital LSI 11/23 minicomputer. Data acquisition was carried out at sampling speeds of up to 500 samples per second and thus enabled establishment of accurate force–deformation and stress–relaxation curves.

RESULTS AND DISCUSSION

The experimental results show (as could have been anticipated) that there is a big difference between the various foam groups. The polyurethane (described in a previous publication[5]) and the polyethylene foams are rate-dependent materials while the expanded PS is rate-independent. Figures 1 and 2 demonstrate this behavior for PE and PS foams, respectively. In the case of rate-independent foams, no model is required for the prediction of their mechanical properties as their behavior depends primarily on their density, as shown in Figure 3. For the rate-dependent PE foams, straight relaxation lines were also obtained when plotting log E_r vs. log t. However, these lines were not parallel when plotted at various initial strains as can be seen in Figure 4. The relaxation modulus in Figure 4 can be described by the following equation:

$$E_r(t_i, \epsilon_i, \dot{\epsilon}_i) = E_{\mathrm{ref}}(t_i)(1/t_i)^{n(\epsilon_i, \dot{\epsilon}_i)} t^{-n(\epsilon_i, \dot{\epsilon}_i)} \tag{10}$$

The reference modulus $E_{\mathrm{ref}}(t_i)$ was chosen for $t_i = 1$ s, namely $E_r(1)$.

By shifting the different lines up or down, they can be brought to all intersect with one reference line (say, the one at $\epsilon = 0.1$ and $t = 1$ s). The

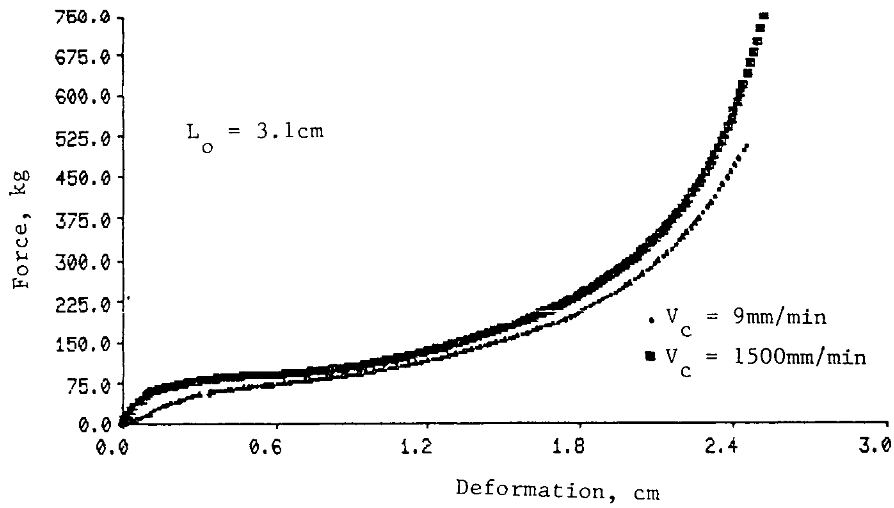

Fig. 1. Effect of compressive strain rate on the force–deformation curves of PE foam.

different lines can then be rotated around this point of intersection so that they will all fall on one master curve. From the amount of upwards or downwards shifting in order to get each relaxation line (obtained at the different initial strains) to the intersection point, the strain function $f(\epsilon)$ was obtained using the same fitting techniques previously described[5]:

$$f(\epsilon) = \sum_{m=0}^{N} A_m \epsilon^m \tag{11}$$

From the amount of rotation around the intersection point required for each

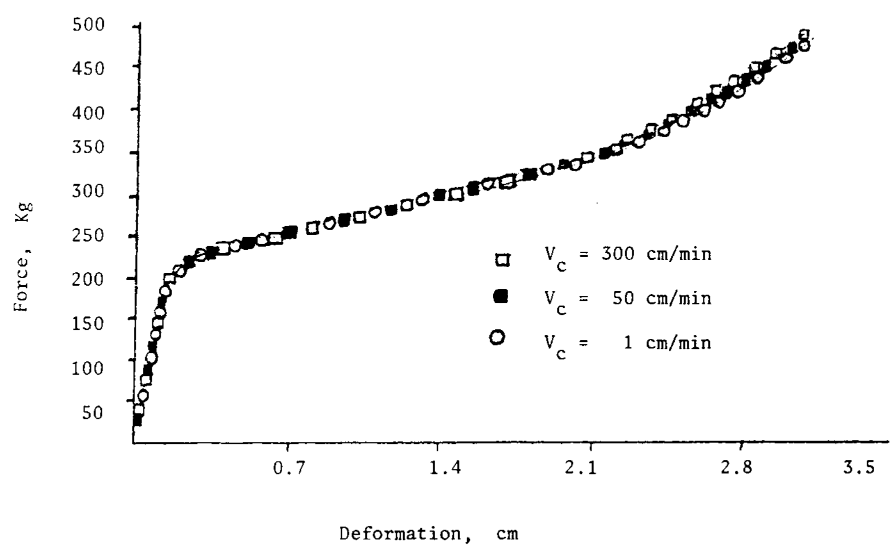

Fig. 2. Effect of compressive strain rate on the force–deformation curves of PS foam.

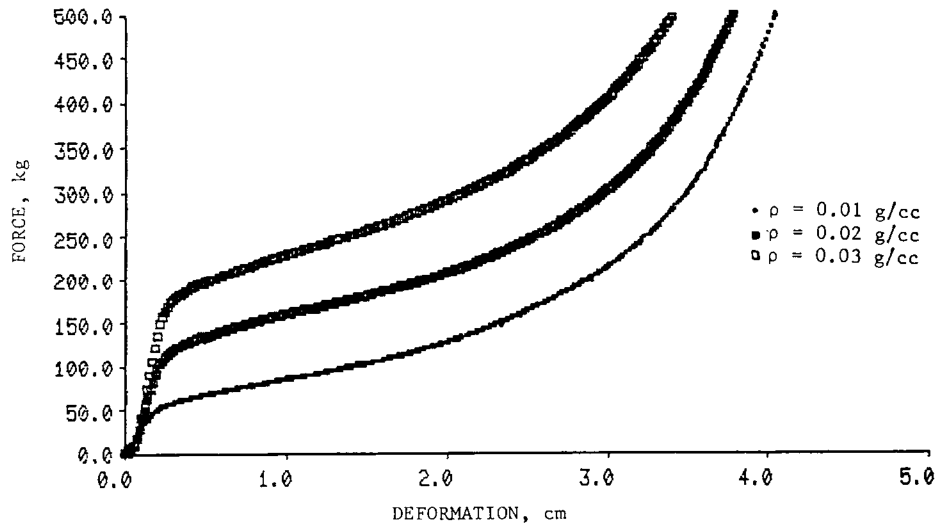

Fig. 3. Effect of foam density on the force–deformation curve of expanded PS.

line to coincide with the reference relaxation line, the constants a and b in eq. (5) could be determined if n is only strain-dependent. However, as will be shown later in this publication, the slope n for the closed cell foams was found to be strain as well as strain-rate-dependent. Thus, eq. (5) is inapplicable for this case.

The following equation was found to describe well this dependence:

$$n(\epsilon, \dot{\epsilon}) = C_0 + C_1\dot{\epsilon} + C_2\dot{\epsilon}^2 + \sum_{m=0}^{N} B_m\epsilon^m \tag{12}$$

A_m and B_m in eqs. (11) and (12) are coefficients.

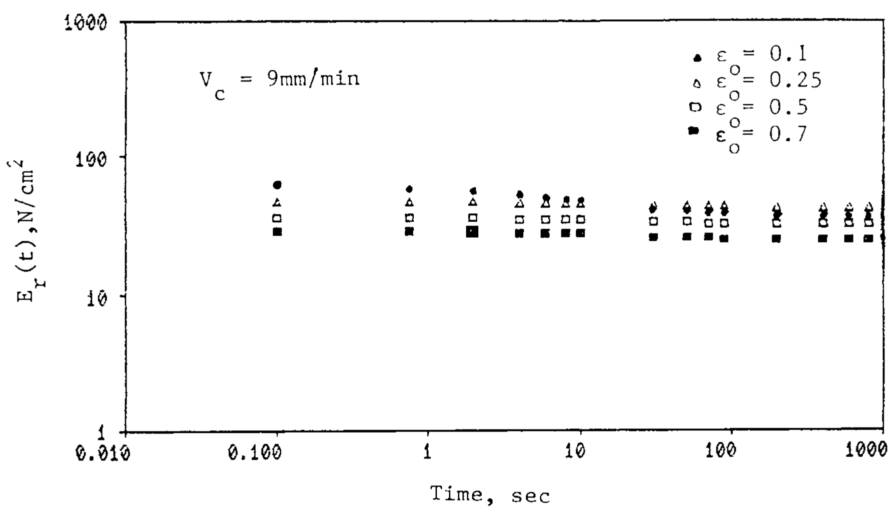

Fig. 4. Stress–relaxation curves of PE foam.

TABLE II
Measured and Calculated Values of the Slope n for the crosslinked PE Foam (at One Stain Rate)[a]

Compressive strain	$-n$ (exptl)	$-n$ (calcd)
0.05	0.049	0.048
0.10	0.039	0.040
0.20	0.031	0.030
0.25	0.027	0.027
0.30	0.023	0.023
0.40	0.019	0.019
0.50	0.016	0.016
0.60	0.013	0.013
0.70	0.011	0.011
0.75	0.010	0.010

[a] $L_\epsilon = 3.0$ cm and $\epsilon_0 = 0.0050$ s^{-1}.

In both eqs. (11) and (12) only five terms of summation were required in order to describe well the effect of the strain on the strain function and the slope n. The calculated [from eq. (12)] and measured slope n are compared in Table II at one strain rate. Similar results were obtained at different strain rates and a very good agreement was found.

The solution of the modified Boltzman integral [eq. (7)] results in the following constitutive equation:

$$\sigma(\epsilon,\dot{\epsilon}) = K_r f(\epsilon)\frac{1}{1-n(\epsilon,\dot{\epsilon})}\dot{\epsilon}^{n(\epsilon,\dot{\epsilon})}\epsilon^{1-n(\epsilon,\dot{\epsilon})} \tag{13}$$

A numerical method was used to calculate the effect of strain and strain rate on the stress according to eq. (13). It is clear that foams exhibiting a strain and strain rate dependency of the slope n require a larger number of experiments for their characterization than a foam for which n is strain-dependent only or strain rate only. However, after the functions $n(\epsilon,\dot{\epsilon})$ and $f(\epsilon)$ have been determined, good agreement between the calculated and experimentally determined curves are obtained as demonstrated in Figure 5 for the PE foam. It was shown in an earlier publication[3] that the stress–strain curves, at any compression rate, can be determined from a reference stress–strain curve determined at a low compression rate and parameters obtained from a limited number of stress–relaxation experiments (the reference model). The calculated stress $\sigma_i(\epsilon,\dot{\epsilon})$, according to this reference model, is

$$\sigma_i(\epsilon,\dot{\epsilon}) = \sigma_0\frac{K_{r,i}}{K_{r,0}}\cdot\frac{1-n_0\cdot\dot{\epsilon}_i^{n_i}}{1-n_i\cdot\dot{\epsilon}_0^{n_0}}\epsilon^{n_0-n_i} \tag{14}$$

where the index i and zero refer to any strain rate and the reference strain rate, respectively. When K_r and $n(\epsilon,\dot{\epsilon})$ do not depend on the strain rate,

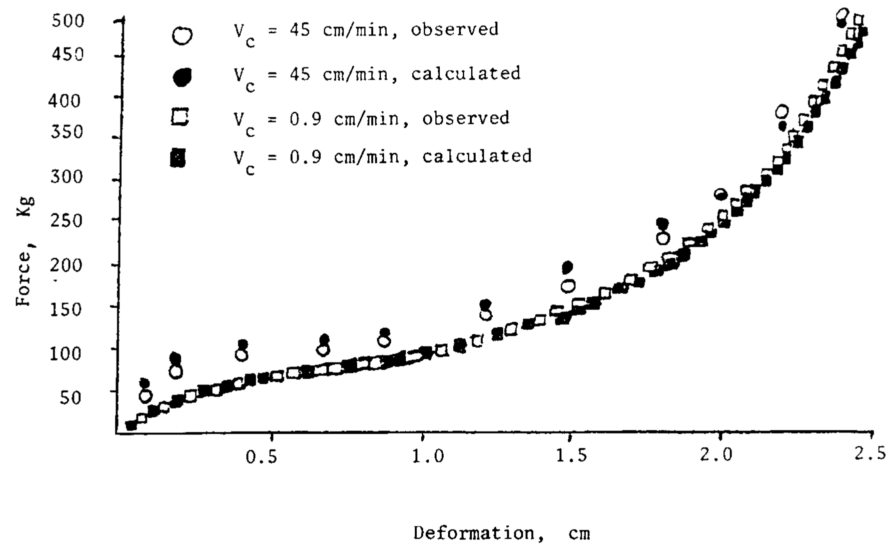

Fig. 5. Measured and calculated force-deformation curves for PE foam using the modified Boltzman integral.

eq. (14) is reduced to an expression very similar to the one given by Nagy et al.[10]:

$$\sigma_i = \sigma_0 \left(\dot{\epsilon}_i / \dot{\epsilon}_0 \right)^{n(\epsilon)} \tag{15}$$

which is based on the assumption that n is linearly dependent on the strain [see eq. (5)].

In Figure 6 the calculated curves using the reference model [eq. (14)] are compared to the experimentally determined ones for the PE foam. It can be

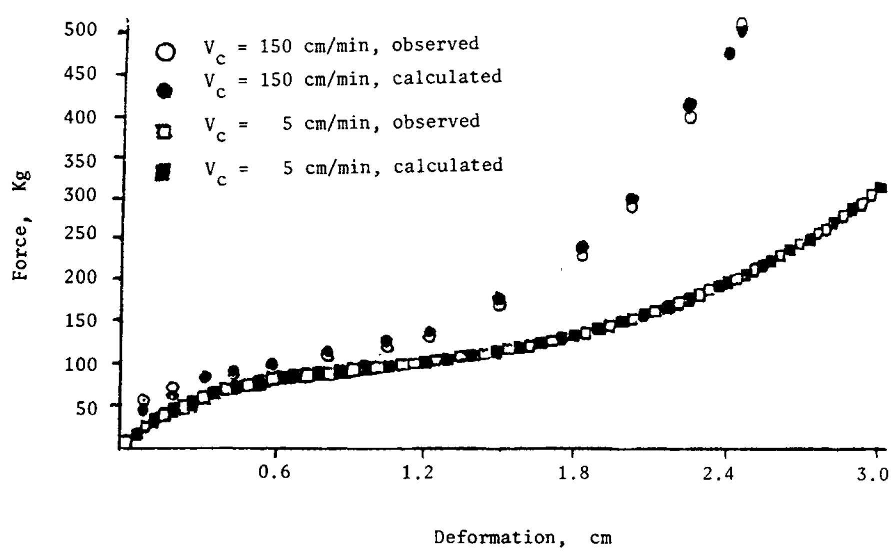

Fig. 6. Measured and calculated force deformation curves for PE foam using our reference model.

TABLE III
Effect of Strain and Strain Rate on the Slope n [eq. (4)] and K_r for the Studied PE Foam[a]

Strain \ Strain rate (s^{-1})	$-n$	$-n$	K_r	K_r
	0.05	0.25	0.05	0.25
0.20	0.031	0.041	—	—
0.25	0.027	0.038	—	—
0.30	0.022	0.036	31.2	32.0
0.50	0.016	0.026	30.5	32.0
0.70	0.011	0.018	39.7	40.6

[a] L_0 = 3.0 cm; foam density = 0.04 g/cc.

seen that the agreement is even better than that obtained from the predicted results using the modified Boltzman integral [eq. (6)].

The Effect of Compression Rate of the Parameters of the Constitutive Equations [eqs. (8) and (14)]

In Figures 1 and 2 it was shown that PE closed cell foams are rate-dependent whereas the PS foams are rate-independent. The effect of strain and strain rate on the parameter n are shown in Table III. The experimental results showed that, for the PS foam, n is strain-rate-independent and very slightly strain-dependent. For the PE foam, on the other hand, n depends both on the strain as well as on the strain rate. The strain function $f(\epsilon)$, calculated by a curve fitting procedure, according to eq. (11), was found to be rate-independent (see Fig. 7) in accordance with previous findings.[3,4] It was also found that the parameter K_r is almost rate-independent as can be seen in Table III.

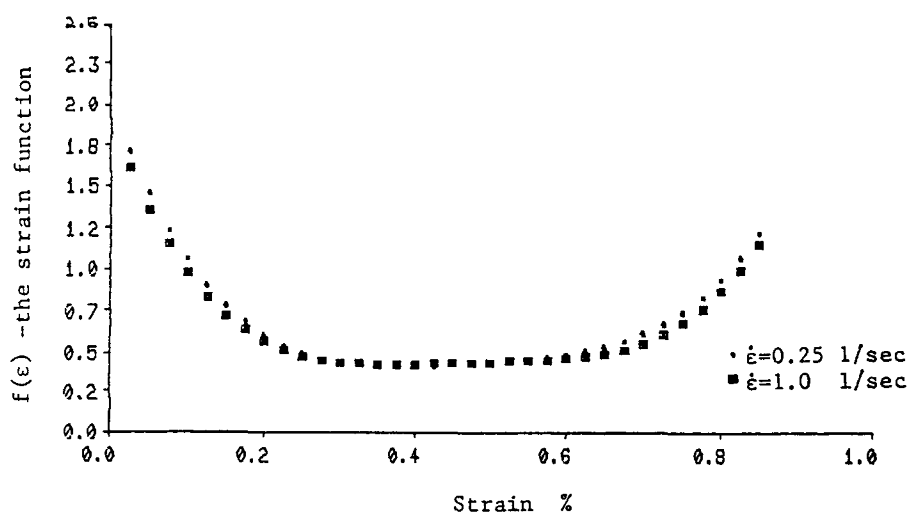

Fig. 7. Effect of compressive strain rate on the strain function $f(\epsilon)$.

To summarize, it was found that the closed cell foams can be divided into two groups, one strain-rate-dependent (the relatively flexible PE foam) and the other strain-rate-independent (the brittle PS foam). For the strain-rate-independent foam the stress–strain curves measured at different strain rates are almost identical. For the strain-rate-dependent foam, two methods were shown to predict the stress–strain curve at any strain rate from a limited number of stress–relaxation experiments and one stress–strain curve determined at a low strain rate. These methods are the modified Boltzman integral method (hereditary integral) and our reference model. The latter method was shown to better predict the experimentally measured stress–strain curves.

References

1. J. Miltz and G. Gruenbaum, *Polym. Eng. Sci.*, **21**, 1010 (1981).
2. G. Gruenbaum and J. Miltz, *J. Appl. Polym. Sci.*, **28**, 135 (1983).
3. O. Ramon, S. Mizrahi, and J. Miltz, *Polym. Eng. Sci.*, in press.
4. K. C. Rusch, *J. Appl. Polym. Sci.*, **13**, 2297 (1969).
5. K. C. Rusch, *J. Appl. Polym. Sci.*, **14**, 1263 (1970).
6. J. T. Tsai, *Polym. Eng. Sci.*, **22**(9), 545 (1982).
7. E. A. Meinecke and R. C. Clark, *Mechanical Properties of Polymeric Foams*, Technomic, Westport, CT, 1973.
8. J. Miltz and O. Ramon, *Polym. Eng. Sci.*, **28**(19), 1305 (1986).
9. S. D. Hong, R. F. Fedors, F. Schwartz, and R. F. Kandel, *Polym. Eng. Sci.*, **21**(11), 688 (1981).
10. A. Nagy, W. L. Ko, and V. S. Lindholm, *J. Cell. Plast.*, May–June, **127** (1974).
11. D. S. Kerstner, NAI-57-187, Northrup Aviation, IAC, Hawthorne, CA, 1957.

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